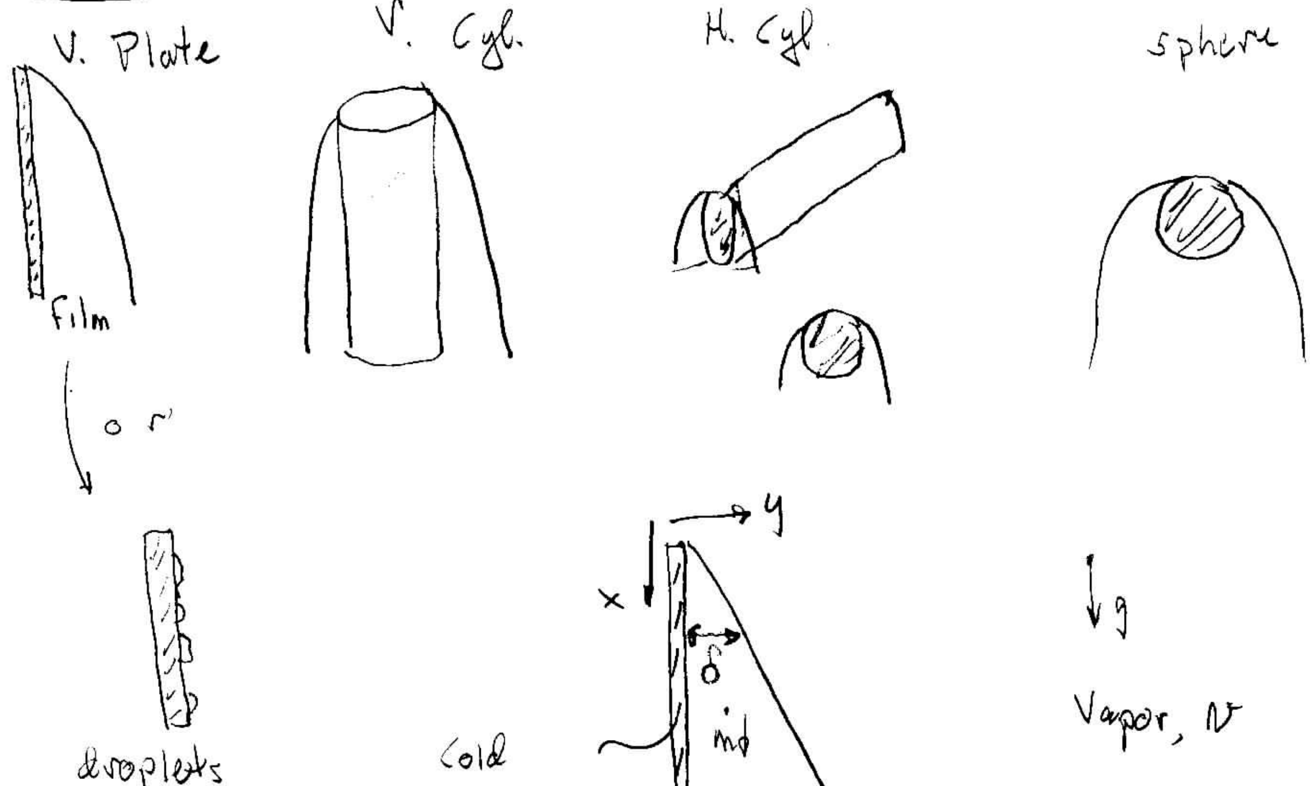
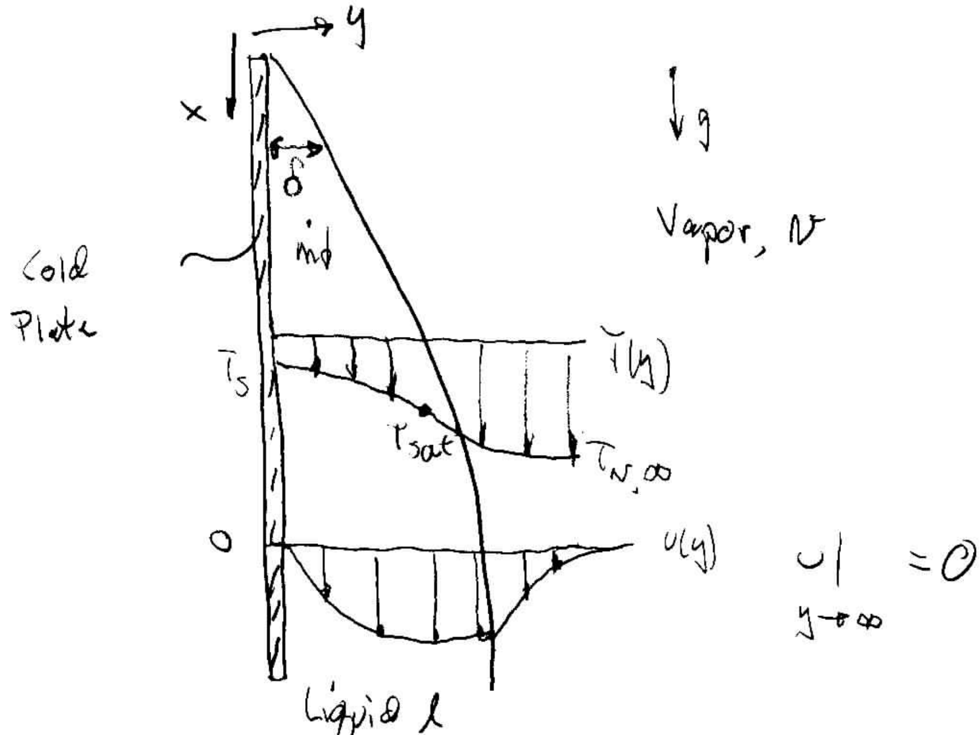
Condensation





Modified heat of vaporization (sub cooled liq.)

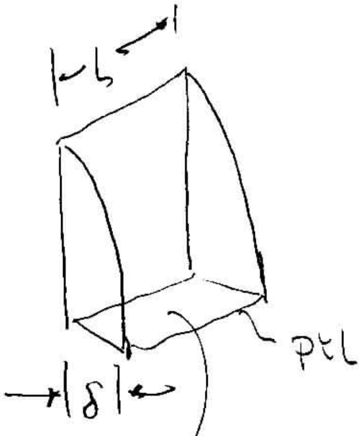
$$h_{fg}^* = h_{fg} + \underbrace{0.68\, c_{p,l} (T_{sat} - T_{surf.})}$$

normal h_{fg}

because often the liquid is cooled below T_{sat} by the cool plate ~ $T_{surf.} < T_{sat.}$

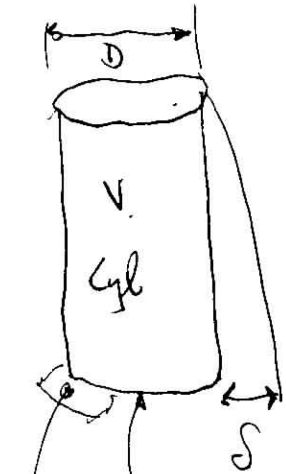
Similar for superheated vapor

$$h_{fg}^* = h_{fg} + 0.68\, c_{p,l} (T_{sat} - T_{sur}) + c_{p,v} (T_v - T_{sat.})$$



$A_c = L\delta$

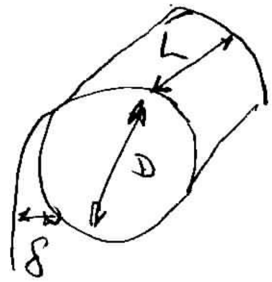
$$D_h = \frac{4A_c}{P} = 4\delta$$



$P = \pi D$

$A_c = \pi D \delta$

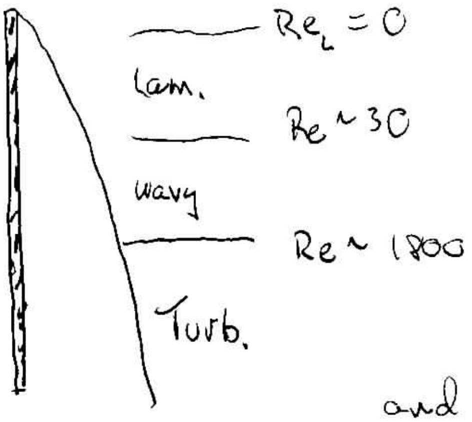
$$D_h = \frac{4A_c}{P} = 4\delta$$



$P = 2L$

$A_c = 2L\delta$

$$D_h = \frac{4A_c}{P} = 4\delta$$



$$Re = \frac{D_h \rho_l V_l}{\mu_l} = \frac{4 A_c \rho_l V_l}{P \mu_l}$$

$$= \frac{4 \rho_l V_l \delta}{\mu} = \frac{4 \dot{m}}{P \mu_l}$$

sub. for $\dot{m}$

and

$$\dot{Q}_{cond.} = h A_{surf} (T_{sat} - T_{surf}) = \dot{m} h_{fg}^*$$

for cond.

heat trans. coef.

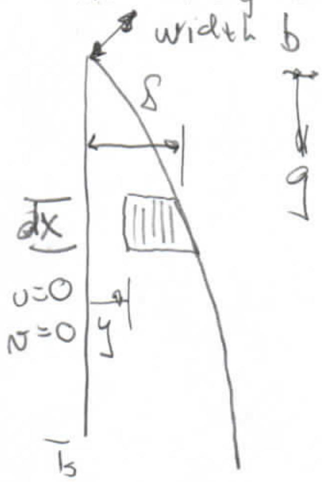
to get

$$Re = \frac{4 \dot{Q}_{cond}}{P \mu_l h_{fg}^*} = \frac{4 A_s h (T_{sat} - T_s)}{P \mu_l h_{fg}^*}$$

fluid prop. at $T_{film} = \frac{(T_{sat} + T_{surf})}{2}$

But h_{fg} is at T_{sat} (not affected by subcooling!)

Same analysis



Assume

- $T_{surf} \sim$ ¢
- Pure cond. across film ($T(x)$ is linear across δ) (no conv.) Big assumption here!
- U_{vapor} is low enough that no drag (visc. shear ~ 0)
- laminar flow in film
- accel. is ~0 in film layer

So

$$\Sigma F_x = m a_x = 0$$

$$\rho_\ell g(\delta - y) b\,dx = \mu_\ell \frac{du}{dy}(b\,dx) + \rho_v g(\delta - y)(b\,dx)$$

or $$\frac{du}{dy} = \frac{g(\rho_\ell - \rho_v)(\delta - y)}{\mu_\ell} \quad \text{with } u\big|_{y=0} = 0$$

or $$u(y) = \frac{g(\rho_\ell - \rho_v)}{\mu_\ell}\left(y\delta - \frac{y^2}{2}\right)$$

Look at

$$\dot{m}(x) = \int_{y=0}^{\delta} \rho_\ell u(y)\, b\, dy = \frac{g b \rho_\ell (\rho_\ell - \rho_v)\delta^3}{3\mu_\ell} \qquad \text{recall } \delta(x)!$$

so that

$$\frac{d\dot{m}}{dx} = \frac{g b \rho_\ell (\rho_\ell - \rho_v)\delta^2}{\mu_\ell}\frac{d\delta}{dx} \qquad (*)$$

But also

$$\underbrace{d\dot{q}}_{\text{heat trans. from vapor to plate through film}} = \underbrace{h_{fg}\,\dot{m}}_{\text{heat released as vapor is condensed}} = k_\ell (b\,dx)\frac{(T_{sat} - T_s)}{\delta}$$

or $$\frac{d\dot{m}}{dx} = \frac{k_\ell b}{h_{fg}}\frac{(T_{sat} - T_s)}{\delta(x)} \qquad (*) \text{ Equate}$$

Equate *s and sep. variables

$$\delta^3 \, d\delta = \frac{\mu_\ell k_\ell (T_{sat} - T_s)}{g \rho_\ell (\rho_\ell - \rho_v) h_{fg}} \, dx \qquad \text{Now } \int_{x=0}^{x} (\ \ ) \, dx$$

to get

$$* \quad \delta(x)^4 = \frac{4 \mu_\ell k_\ell (T_{sat} - T_{surf}) x}{g \rho_\ell (\rho_\ell - \rho_v) h_{fg}}$$

So heat transfer to plate at a location x is

$$q''(x) = h(x)(T_{sat} - T_{surf})$$

$$= k_\ell \frac{(T_{sat} - T_s)}{\delta(x)} \Rightarrow h(x) = \frac{k_\ell}{\delta(x)} \quad * \text{ Put into here}$$

$$h(x)^4 = \frac{g \rho_\ell (\rho_\ell - \rho_v) h_{fg} k_\ell^3}{4 \mu_\ell (T_{sat} - T_{surf}) x} \quad \text{gives local } h(x)$$

Can now find average $\bar{h}_L = \frac{1}{L} \int_{x=0}^{L} h(x) \, dx = \frac{4}{3} h(L)$

$$= 0.943 \left[\frac{g \rho_\ell (\rho_\ell - \rho_v) h_{fg} k_\ell^3}{\mu_\ell (T_{sat} - T_{sur}) L} \right]^{1/4}$$

But, it works better using h^*_{fg} to account for non-linear $T(x)$.

If one assumes $\rho_v \ll \rho_\ell$ so that $\rho_\ell - \rho_v \approx \rho_\ell$ then

$$Re = \frac{4 g \rho_\ell (\rho_\ell - \rho_v) \delta^3}{3 \mu_\ell^2} \approx \frac{4 g \rho_\ell^2}{3 \mu_\ell^2} \left(\frac{k_\ell}{h(L)} \right)^3 = \frac{4g}{3 \nu_\ell^2} \left(\frac{k_\ell}{3 h_{vert}/4} \right)^3$$

↑ solve

$$\boxed{\text{Laminar} \quad h_{\text{vert plate}} \approx 1.47 \, k_\ell \, Re^{-1/3} \left(\frac{g}{\nu_\ell^2} \right)^{1/3} \quad 0 < Re < 30, \quad \rho_v \ll \rho_\ell}$$

Works well

Prop @ $T_{film} = \frac{T_{sat} + T_s}{2}$ but h_{fg}, ρ_v @ T_{sat}

Wavy on | plates

$$h = \frac{Re\, k_\ell}{\left(1.08\, Re^{1.22} - 5.2\right)} \left(\frac{g}{\nu_\ell^2}\right)^{1/3} \qquad 30 < Re < 1800 \qquad \rho_v \ll \rho_\ell$$

Turb on | plates

$$h = \frac{Re\, k_\ell}{8750 + 58\, P_r^{-1/2}\left(Re^{3/4} - 253\right)} \left(\frac{g}{\nu_\ell^2}\right)^{1/3} \qquad Re > 1800 \qquad \rho_v \ll \rho_\ell$$

Horiz. Tubes

$$h_{horiz} = 0.729 \left[\frac{g\,\rho_\ell(\rho_\ell - \rho_v)\, h_{fg}^{*}\, k_\ell^3}{\mu_\ell\,(T_{sat} - T_{sur})\, D}\right]^{1/4} \qquad \left[\frac{W}{m^2 K}\right]$$

Vert Tubes

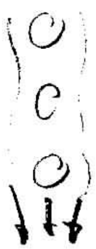
$$\frac{h_{vert}}{h_{horiz}} = 1.29\left(\frac{D}{L}\right)^{1/4}$$

Note if $\frac{h_v}{h_h} = 1 \Rightarrow L = 2.77\,D$

If tube $L = 2.77\,D$ you get same h_{vert} & h_{horiz}.

In practice... $L > 3D$ so $\boxed{\text{use horiz. tubes}}$

Tube Banks



$$h_{hori,\,N\,tubes} = 0.729 \left[\frac{g\,\rho_\ell(\rho_\ell - \rho_v)\, h_{fg}^{*}\, k_\ell^3}{\mu_\ell\,(T_{sat} - T_{surf})\, N\, D}\right]^{1/4} = \frac{h_{horiz,\,1\,tube}}{N^{1/4}}$$

Ex Vert. Plate

1atm vapor (Sat. Steam)

$A_s = W \times L = 6\,m^2$

Calc
- rate of condensation to plate
- rate at which condensate drips of bottom

Assume
(SS)
- $T_{plate} = $ ¢ (t and x)
- Wavy flow (verify this!)
- $\rho_v << \rho_l$

Prop.

Steam

$T_{sat} = 100°C$

$h_{fg} = 2257 \times 10^3 \frac{J}{kg}$

$\rho_v = 0.6 \frac{kg}{m^3}$

Water

$T_{film} = \frac{(T_{sat} + T_s)}{2} = \frac{(100+80)°C}{2} = 90°C$

$\rho_l = 965.3 \frac{kg}{m^3}$ $\quad C_{p,l} = 4206\ J/kg\,K$

$\mu_l = 0.315 \times 10^{-3} \frac{kg}{ms}$ $\quad k_l = 0.675 \frac{W}{mK}$

$\nu_l = 0.326 \times 10^{-6} \frac{m^2}{s}$

Calculations

Modified $h^*_{fg} = h_{fg} + 0.68\, C_{p,l}(T_{sat} - T_s) = 2314 \times 10^3\ J/kg$

For wavy flow
$$Re = \left[4.81 + \frac{3.70\, L\, k_l\,(T_{sat} - T_s)}{\mu_l\, h^*_{fg}}\left(\frac{g}{\nu_l^2}\right)^{1/3}\right]^{0.82} = 1287$$

so $30 < Re < 1800 \Rightarrow$ Wavy

Now
$$h = h_{\substack{vert\\wavy}} = \frac{Re\, k_l}{(1.08\, Re_l^{1.22} - 5.2)}\left(\frac{g}{\nu_l^2}\right)^{1/3} = 5850\ W/m^2K$$

$$q = hA_s\,(T_{sat} - T_s) = 7.02 \times 10^5\ W$$

finally
$$\dot{m}_{condens.} = \frac{q}{h^*_{fg}} = 0.303\ kg/s$$

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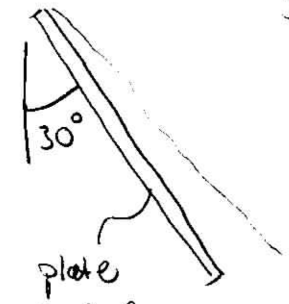
Ex| Condensation on Tilted Plate (Previous prob but tilted)



Steam 100°C

g

Replace g with $g\cos\theta$

$$h_{tilt} = h_{vert} (\cos\theta)^{1/4}$$

h_{vert}: $5850 \frac{W}{m^2 K}$

$$h_{tilt} = 5643 \; W/m^2 K$$

Then

$$q = h A_s (T_{sat} - T_s)$$

A_s: $6 \, m^2$

$$= 6.77 \times 10^5 \; W$$

and

$$\dot{m}_{conden} = \frac{q}{h^*_{fg}} = 0.293 \; kg/s$$

h^*_{fg}: $2314 \times 10^3 \; J/kg$

About 3.3% decrease with tilted plate.

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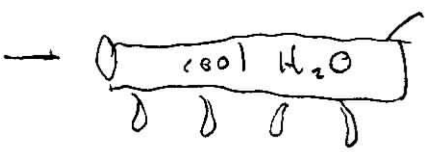
Ex 1 Horiz. tube

Sat. steam 40 °C, P = 7,38 kPa



Pipe D = 3 cm

$A_s = \pi D L = 0{,}09425\ m^2$

Calc
- Rate of heat transfer to cooling H_2O
- Rate of condensation per unit length

Assume (SS) Tube is isothermal

Prop. Sat. Steam P = 7,38 kPa T = 40 °C

$$h_{fg} = 2407 \times 10^3\ J/kg$$

$$\rho_v = 0{,}05\ kg/m^3$$

Water $T_{film} = \frac{T_{sat} + T_s}{2} = \frac{(40+30)°C}{2} = 35°C$

$\rho_l = 994\ kg/m^3$ $\quad c_{p,l} = 4178\ J/kg\,K$

$\mu_l = 0{,}720 \times 10^{-3}\ \frac{kg}{m\,s}$ $\quad k_l = 0{,}623\ W/mK$

Calculations Modified $h^*_{fg} = h_{fg} + 0{,}68\, c_{p,l} (T_{sat} - T_s) = 2435 \times 10^3\ J/kg$

Since $\rho_v << \rho_l$

$$h_{horiz} = 0{,}729 \left[\frac{g \rho_l \overbrace{(\rho_l - \rho_v)}^{\approx \rho_l} h^*_{fg} k_l^3}{\mu (T_{sat} - T_s) D} \right]^{1/4} = 9294\ W/mK$$

$$q_{condens.} = h A_s (T_{sat} - T_s) = 8760\ W$$

$$\dot{m}_{conden.} = \frac{q}{h^*_{fg}} = 0{,}0036\ kg/s$$

So about 3,6 g/s ~13 $\frac{kg}{h}$ per meter of pipe length.

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